# EGC220 Class Notes 2/28/2023

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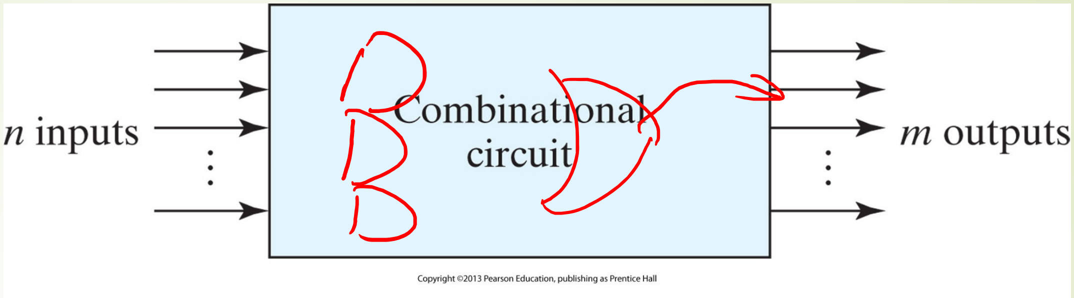
# Test 1:

- Number systems
  - Convert any base to any base
  - Quick conversion between base 2, 4, 8, and 16
  - Add, subtract, multiply in any base
- Logic gate implementation of a Boolean function
- Boolean properties and laws
- Simplification of Boolean algebra using Boolean laws
- Representing Boolean functions in terms of
  - Sum of min-terms, product of max-terms, standard sum of products, standard product of sums, minimum sum of products, minimum product of sums
  - Representing Boolean functions using all NAND or NOR gates.
- Simplification using K-map (up to 5 variables)
  - SOP, POS, Standard SOP and POS, Min. SOP and POS
- Design of combinational circuits

# Design Steps




- From the specification of the circuit, determine the number of inputs and outputs and assign a symbol to each.
- Derive a truth table, assigning inputs to the left and outputs to the right.
- Place all possible combination of inputs i.e all 0's to all 1's
- Using the problem definition determine each output.
- If combinations are left without a specified output, mark them as don't cares.
- Simplify each output using K-map.
- Draw a circuit for each output per requirement ie. All NAND, all NOR, AND – OR, OR – AND, XOR, etc.

Problem 1

Design a circuit that counts the number of 0's present in 4 inputs A, B, C and D. Its output is a multi-bit, representing that count in binary. For example, 0101 has two zeros and therefore the output should be a binary representing 2.

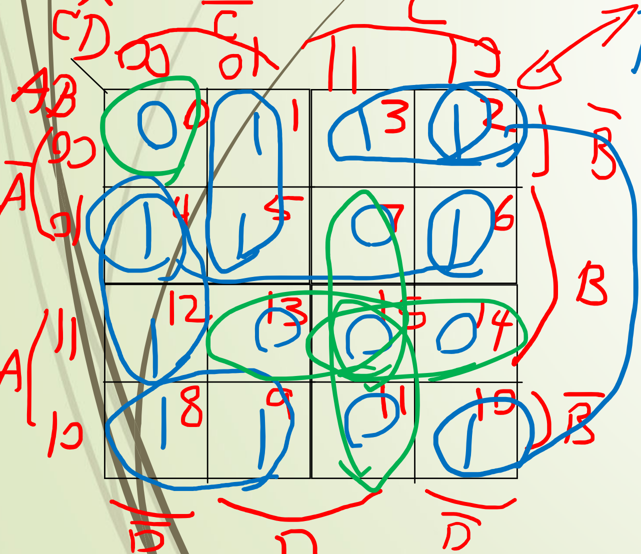
a. Write the truth table for this circuit.
b. Find the minimized logic equations in SOP and POS for each output
c. Draw the corresponding all NAND and all NOR gates logic diagram for this circuit. Label all inputs and outputs.

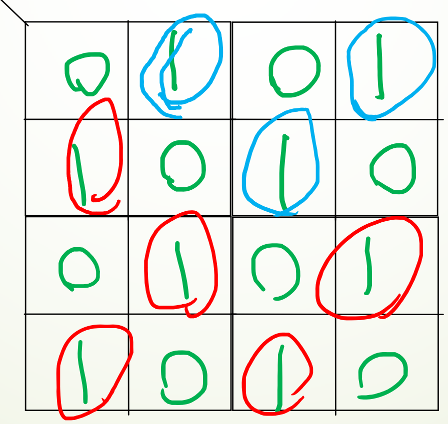
| A | B | C | D | X | Y | Z |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 1 | 1 |
| 0 | 0 | 1 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 1 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 0 |
| 0 | 1 | 1 | 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 0 | 0 | 1 | 1 |
| 1 | 0 | 0 | 1 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 0 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 0 | 0 | 0 |

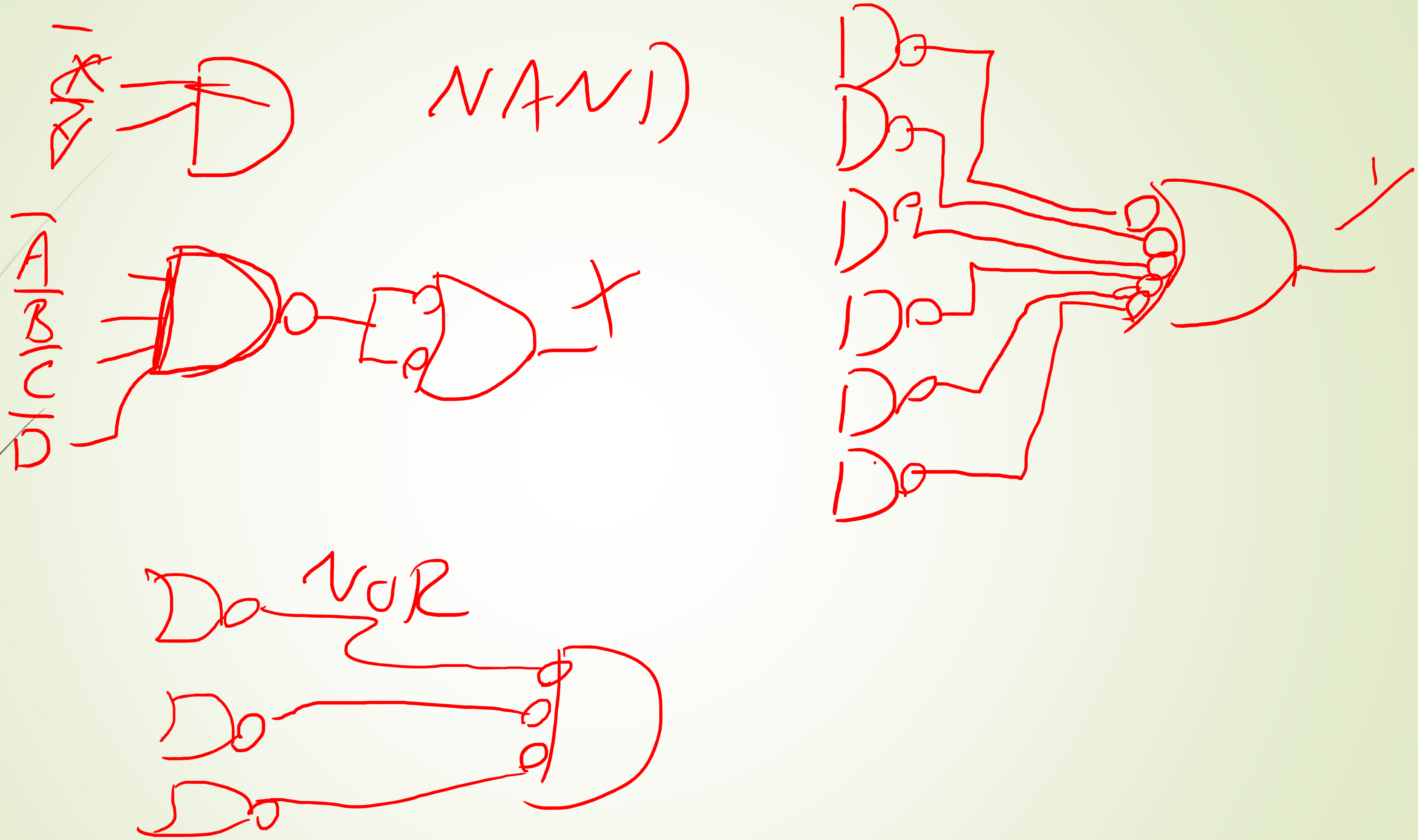
$X = \bar{A}\bar{B}\bar{C}\bar{D}$

$Y = \bar{A}\bar{B}\bar{C} + \bar{B}C\bar{D} + B\bar{C}\bar{D} + \bar{A}B\bar{D} + \bar{A}\bar{C}D + \bar{A}\bar{B}C$

$Z = \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}C\bar{D} + \bar{A}B\bar{C}\bar{D} + \bar{A}BCD + AB\bar{C}D + A\bar{B}CD + ABC\bar{D} + A\bar{B}\bar{C}\bar{D}$






NAND
A
B
C
D
Y
NOR

Problem 1 Design a circuit that counts the number of 0's present in 4 inputs A, B, C and D. Its output is a multi-bit, representing that count in binary. For example, 0101 has two zeros and therefore the output should be a binary representing 2.

a. Write the truth table for this circuit.

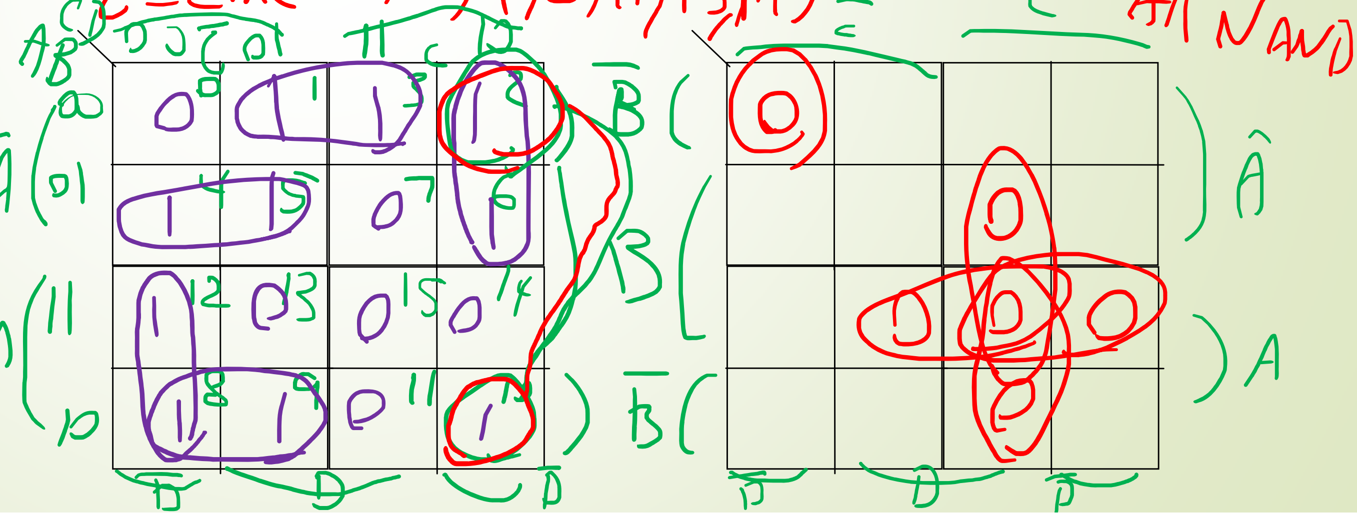
b. Find the minimized logic equations in SOP and POS for each output

c. Draw the corresponding all NAND and all NOR gates logic diagram for this circuit. Label all inputs and outputs.

$X = \bar{A}\bar{B}\bar{C}\bar{D}$

$Y = \Pi M(0,7,11,13,14,15)$

$Z = \Sigma m(1,2,4,7,8,11,13,14)$



$Z = \Sigma m(1, 2, 4, 7, 8, 11, 13, 14)$

✓ All NAND

$Z = \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}C\bar{D} + \bar{A}B\bar{C}\bar{D} + \bar{A}BCD + A\bar{B}\bar{C}\bar{D} + A\bar{B}CD + AB\bar{C}D + ABC\bar{D}$

| | | | |
|---|---|---|---|
| 0 | 1 — 1 | 3 | 1 — 2 |
| 1 — 4 | 5 | 1 — 7 | 6 |
| 12 | 1 — 13 | 15 | 1 — 14 |
| 1 — 8 | 9 | 1 — 11 | 10 |

| | | | |
|---|---|---|---|
| | | | |
| | | | |
| | | | |
| | | | |

All NOR

$\bar{Z} = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}CD + \bar{A}B\bar{C}D + \bar{A}BC\bar{D} + A\bar{B}\bar{C}D + A\bar{B}C\bar{D} + AB\bar{C}\bar{D} + ABCD$

$Z = (A+B+C+D)(A+B+\bar{C}+\bar{D})(A+\bar{B}+C+\bar{D})(A+\bar{B}+\bar{C}+D)(\bar{A}+B+C+\bar{D})$
$(\bar{A}+B+\bar{C}+D)(\bar{A}+\bar{B}+C+D)(\bar{A}+\bar{B}+\bar{C}+\bar{D})$

Problem 1

Design a circuit that counts the number of 0's present in 4 inputs A, B, C and D. the input is BCD. Its output is a multi-bit, representing that count in binary. For example, 0101 has two zeros and therefore the output should be a binary representing 2.

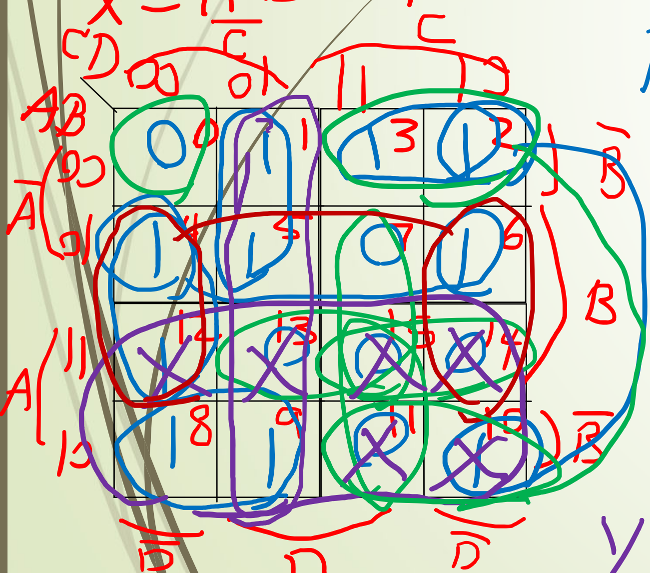
a. Write the truth table for this circuit.
b. Find the minimized logic equations in SOP and POS for each output
c. Draw the corresponding all NAND and all NOR gates logic diagram for this circuit. Label all inputs and outputs.

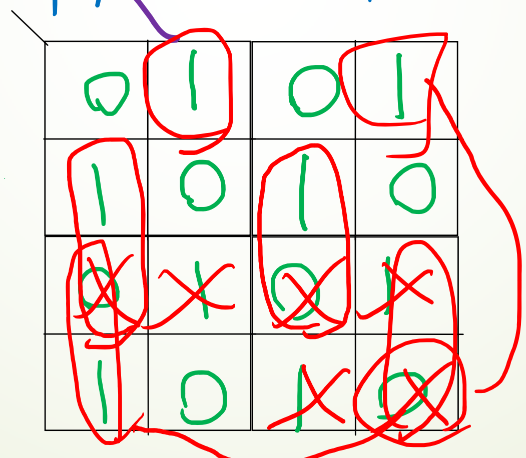
0–9 → BCD

| A | B | C | D | X | Y | Z |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 1 | 1 |
| 0 | 0 | 1 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 1 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 0 |
| 0 | 1 | 1 | 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 0 | 0 | 1 | 1 |
| 1 | 0 | 0 | 1 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | X | X | X |
| 1 | 0 | 1 | 1 | X | X | X |
| 1 | 1 | 0 | 0 | X | X | X |
| 1 | 1 | 0 | 1 | X | X | X |
| 1 | 1 | 1 | 0 | X | X | X |
| 1 | 1 | 1 | 1 | X | X | X |

$Y = \prod M(0, 7, 11, 13, 14, 15)$

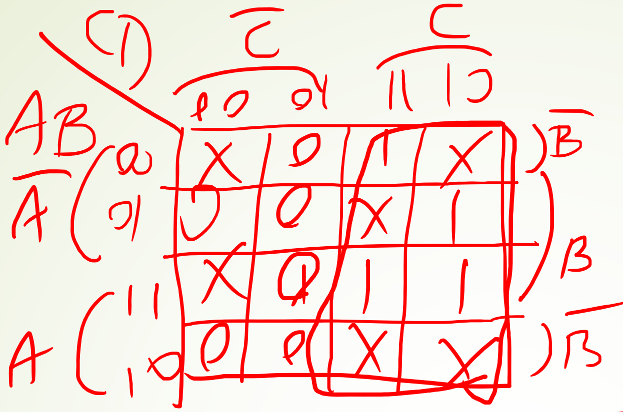
$X = \bar{A}\bar{B}\bar{C}\bar{D}$





$Y = A + \bar{C}D + B\bar{D} + \bar{B}C$

$Z = \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}C\bar{D} + \bar{A}B\bar{C}\bar{D} + \bar{A}BCD + A\bar{B}\bar{C}\bar{D} + A\bar{B}CD + ABC\bar{D} + AB\bar{C}D$




Min. S.O.P

$F = C$

All NAND

C ———— F ✓

Problem 2

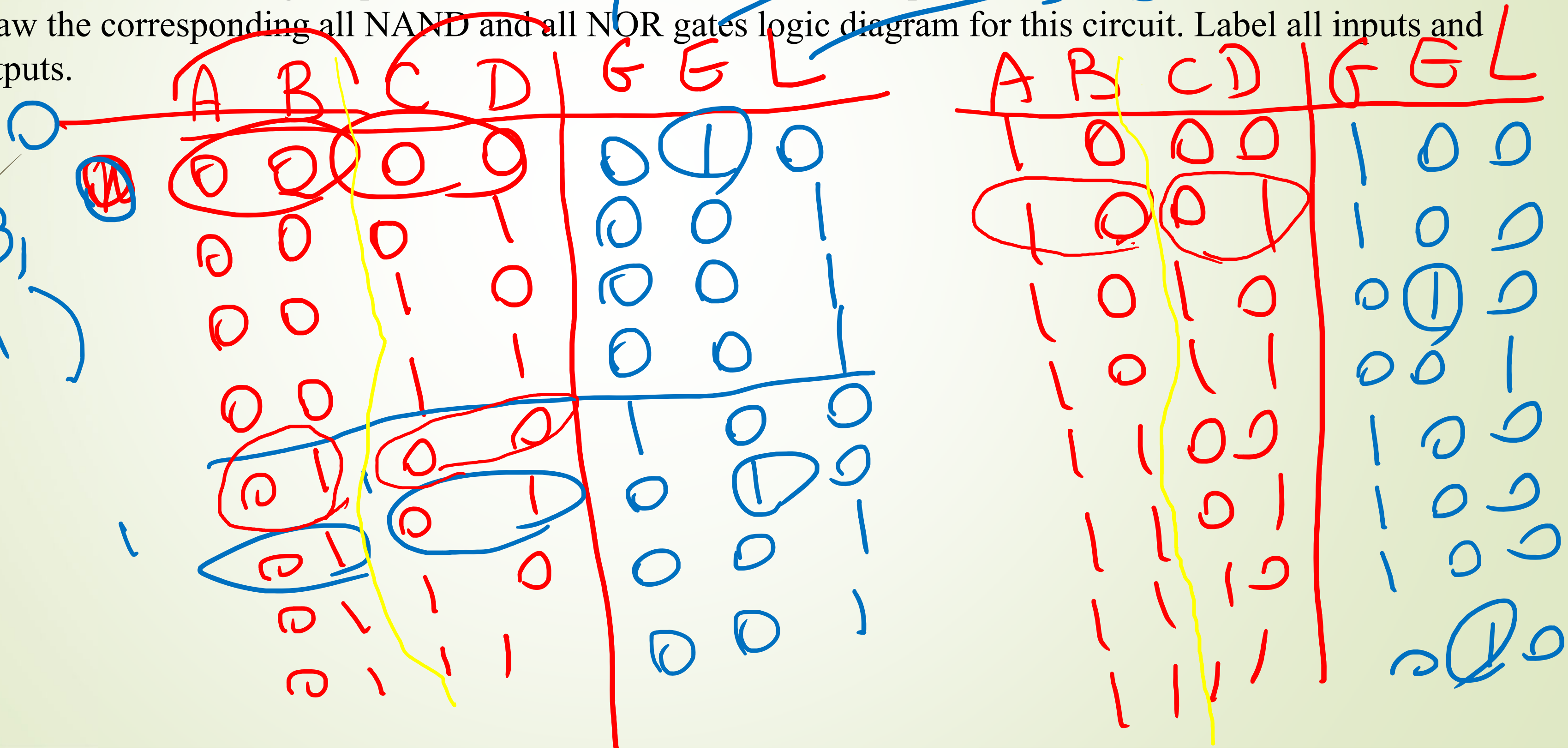
$G = \Sigma m(4, 8, 9, 12, 13, 14)$ $E = \Sigma m(0, 5, 10, 15)$

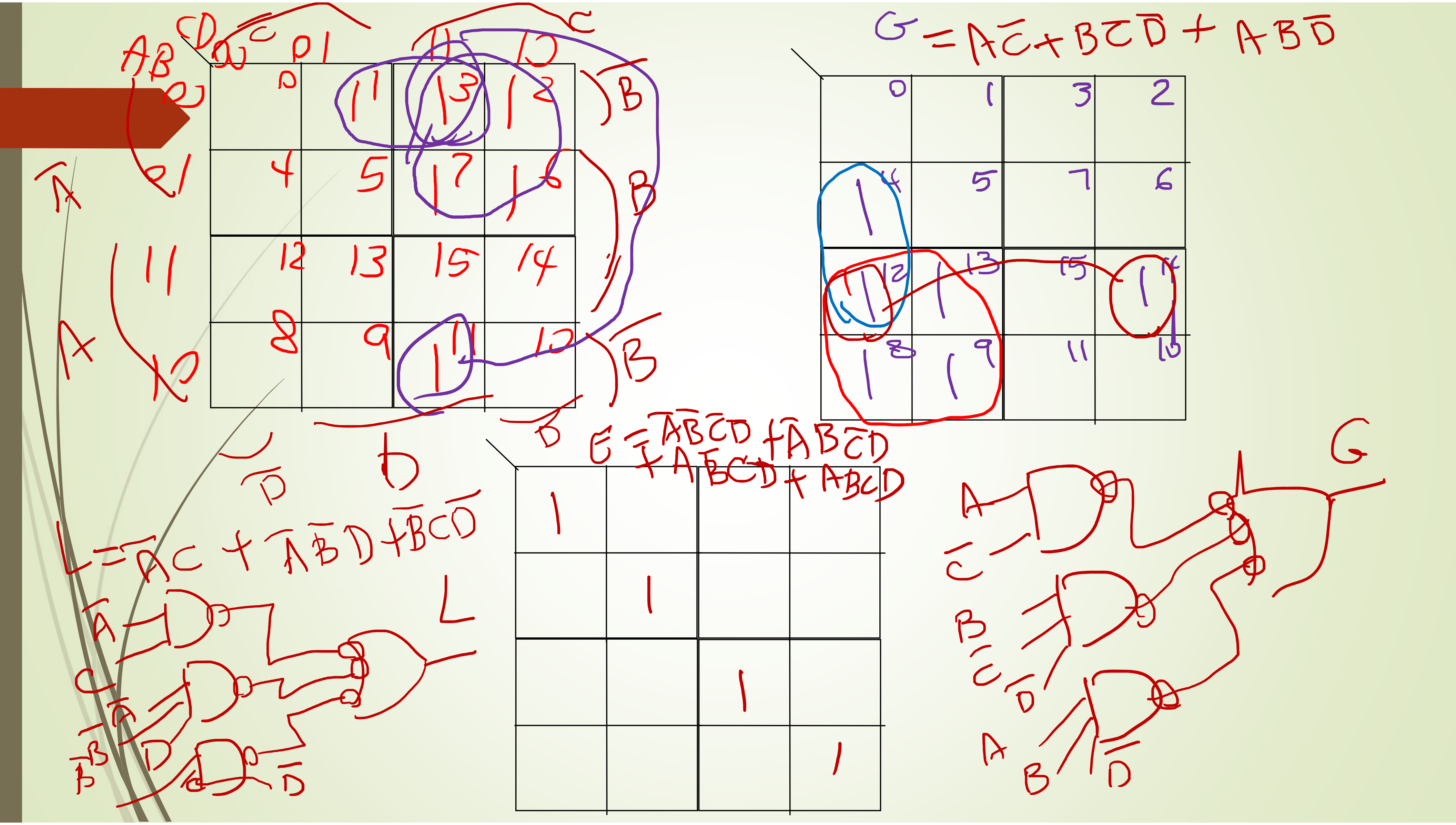
Design a circuit with inputs A,B,C, and D. Let the two inputs AB represent a two-bit number with A as the high order bit, and CD represent another two-bit number. That is, the values on AB represent four values 00 (0), 01 (1), 10 (2), and 11 (3). The circuit has three outputs: G, E, and L. Output G, E, and L should be 1 only if the number represented by AB is greater, equal, and less than the number represented by CD, respectively.

a. Write the truth table for this circuit.
b. Find the minimized logic equations in SOP and POS for each output
c. Draw the corresponding all NAND and all NOR gates logic diagram for this circuit. Label all inputs and outputs.

AB>CD AB=CD AB<CD

$L = \Sigma m(1, 2, 3, 6, 7, 11)$





$G = A\bar{C} + B\bar{C}\bar{D} + AB\bar{D}$
$E = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}B\bar{C}D + A\bar{B}C\bar{D} + ABCD$
$L = \bar{A}C + \bar{A}\bar{B}D + \bar{B}C\bar{D}$

3. Design a circuit that can convert a BCD code into a Gray. a. Write the truth table for this circuit. b. Find the minimized logic equations in SOP and POS for each output c. Draw the corresponding all NAND and all NOR gates logic diagram for this circuit. Label all inputs and outputs.

| 1 bit |
|---|
| 0 |
| 1 |

| 2 bit |
|---|
| 00 |
| 01 |
| 11 |
| 10 |

| 3 bit | |
|---|---|
| 0 | 000 |
| 1 | 001 |
| 2 | 011 |
| 3 | 010 |
| 4 | 110 |
| 5 | 111 |
| 6 | 101 |
| 7 | 100 |

# CKT to convert 2 bit binary to Gray code

Binary → AB, Gray → XY

| A | B | X | Y |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 |

0
1

$X = A$

$Y = A \oplus B$

## Gray code to binary

| X | Y | A | B |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 |

$A = X$

$B = X \oplus Y$

| Binary A | B | C | Gray | | |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 | 0 |
| 1 | 0 | 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 0 | 0 |

$G = A$

# GRAY Code

| | |
|---|---|
| 0 | 0 |
| 0 | 1 |
| 1 | 1 |
| 1 | 0 |

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 0 | 1 | 0 |
| 1 | 1 | 0 |
| 1 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 0 | 0 |

$$Y = \Sigma m(1,2,5,6,9) + d(\ )$$

# BCD → GRAY

BCD

| ABCD | GREY | |
|---|---|---|
| 0000 | 0000 | ← 0 |
| 0001 | 0001 | ← 1 |
| 0010 | 0011 | ← 2 |
| 0011 | 0010 | |
| 0100 | 0110 | |
| 0101 | 0111 | |
| 0110 | 0101 | |
| 0111 | 0100 | |
| 1000 | 1100 | |
| 1001 | 1101 | |
| 1010 | 1111 | don't cares |
| 1011 | 1110 | |
| 1100 | 1010 | |
| 1101 | 1011 | |
| 1110 | 1001 | |
| 1111 | 1000 | ← 15 |

$$G(A,B,C,D) = \Sigma m(8,9) + d(10,11,12,13,14,15)$$

$$R(A,B,C,D) = \Sigma m(4,5,6,7,8,9) + d(\ ``\ )$$

$$E(A,B,C,D) = \Sigma m(2,3,4,5) + d(\ ``\ )$$

BCD → GRAY

| ABCD | GREY |
|---|---|
| 0000 | 0000 |
| 0001 | 0001 |
| 0010 | 0011 |
| 0011 | 0010 |
| 0100 | 0110 |
| 0101 | 0111 |
| 0110 | 0101 |
| 0111 | 0100 |
| 1000 | 1100 |
| 1001 | 1101 |
| 1010 | 1111 |
| 1011 | 1110 |
| 1100 | 1010 |
| 1101 | 1011 |
| 1110 | 1001 |
| 1111 | 1000 |

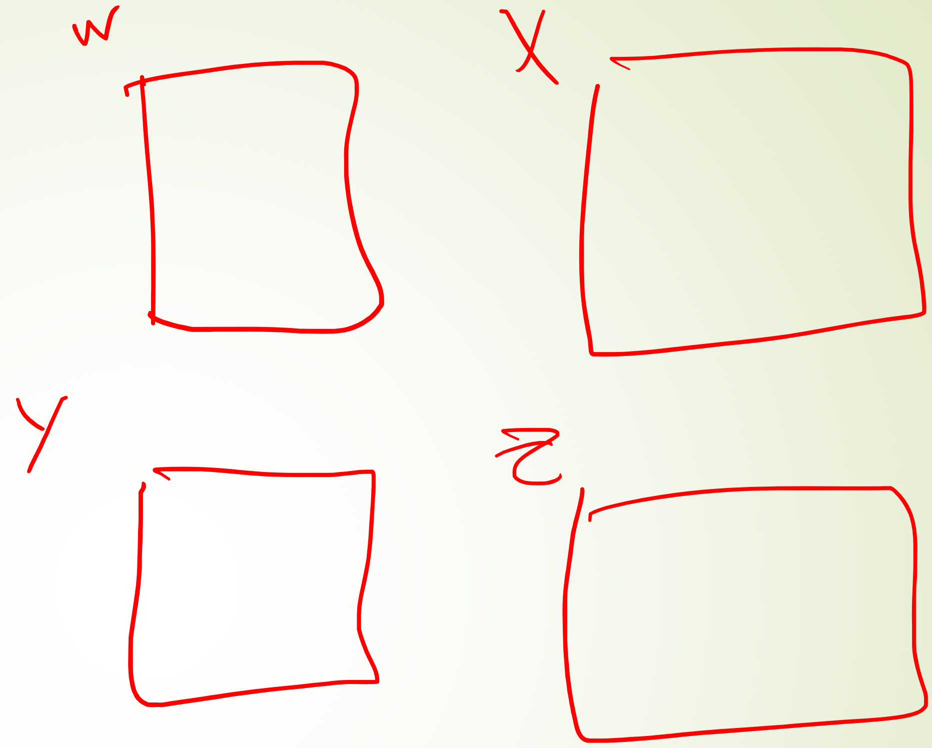
GRAY → BCD

| GREY | ABCD |
|---|---|
| 0000 | 0000 |
| 0001 | 0001 |
| 0010 | 0011 |
| 0011 | 0010 |
| 0100 | 0111 |
| 0101 | 0110 |
| 0110 | 0100 |
| 0111 | 0101 |
| 1000 | XXXX |
| 1001 | XXX1 |
| 1010 | XXXX |
| 1011 | XXXX |
| 1100 | 1000 |
| 1101 | |
| 1110 | |
| 1111 | |

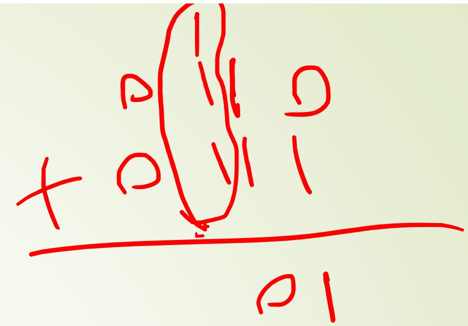
A = Σm( )

B = Σm( — — )

| ABCD | WXYZ |
|---|---|
| 0000 | 0000 |
| 0001 | 0001 |
| 0010 | 0011 |
| 0011 | 0010 |
| 0100 | 0110 |
| 0101 | 0111 |
| 0110 | 0101 |
| 0111 | 0100 |
| 1000 | 1100 |
| 1001 | 1101 |
| 1010 | 1111 |
| 1011 | 1110 |
| 1100 | 1010 |
| 1101 | 1011 |
| 1110 | 1001 |
| 1111 | 1000 |



# FULL ADDER

| A | B | Cin | Sum | Cout |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 |

$$Sum = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC$$

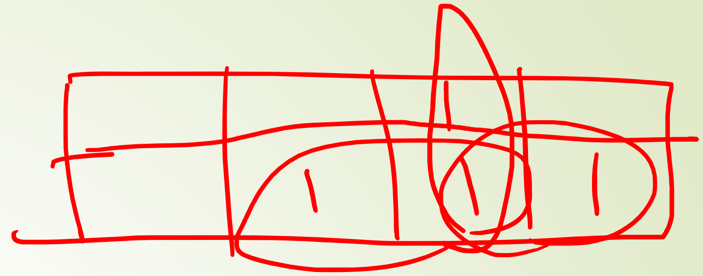
$$\bar{A}(\bar{B}C + B\bar{C}) + A(\bar{B}\bar{C} + BC)$$

$$\bar{A}(\underbrace{B \oplus C}_{X}) + A(\underbrace{\overline{B \oplus C}}_{\bar{X}})$$

$Sum = \bar{A}X + A\bar{X}$

$= A \oplus X$

$= A \oplus B \oplus C_{in}$



K-MAP

$C_{out} = AB + BC_{in} + AC_{in}$

$C_{out} = \bar{A}BC + A\bar{B}C + AB\bar{C} + ABC$

$C(A \oplus B) + AB$